Group theory B.Sc - II Dr Shalini Kumari

Introduction Group.

Algebraic structure

A non empty set with one and more binary operation defined over it and satisfying certain laws of binary operation is called Algebraic structure or algebraic System.

Groupoid Let G be any non empty set and o is a binary operation, then the structure (G, o) is called a Groupoid or quasi Group if $a * b \in G$, $\forall a, b \in G$ i.e. G is closed for the binary operation.

If the binary operation 'o' in the set G satisfies the commutative property is $a o b = b o a$ $\forall a, b \in G$ then, the structure (G, o) is said to be a commutative Groupoid.

The structure (N, +), (N, ·), (Z, +) (Z, ·), (Q, +), (Q, ·) (R, +), (R, ·)

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$(C,+)$, $(C,\cdot)$ are all commutative Groupoids

Note Here
N denotes set of Natural number
Z denotes set of integers
Q denotes set of Rational number
R stands for set of Real numbers
And C denotes set of Complex numbers.

Semi-Group :- Let G be a non-empty set and '$\circ$' be a binary operation defined on it, then the structure $(G,\circ)$ is said to be a Semi Group if
$$a\circ(b\circ c) = a\circ(b\circ c) \quad \forall\, a,b,c \in$$

The structure $(N,+)$ $(N,\cdot)$ $(Z,+)$, $(Z,\cdot)$, $(Q,+)$, $(Q,\cdot)$, $(R,+)$ $(R,\cdot)$, $(C,+)$ $(C,\cdot)$ are commutative Semi-Group

The structure $(Z, -)$, $(Q, -)$ $(R, -)$, $(C, -)$ are not semi-group since associative law is not satisfied

Monoid – Let G a non-empty set and $*$ be a binary operation defined on it. Then the structure $(G, *)$ is said to be a monoid if the following axioms are satisfied.

(I) Associative law $a*(b*c) = (a*b)*c$ $\forall\, a, b, c \in G$.

(II) Existence of identity – There exist an element, denoted by e and called the identity in G such that

$$a*e = a = e*a \quad \forall\, a \in G$$

(i) The structure $(Z, +)$, $(Z, \cdot)$, $(Q, +)$ $(R, +)$ $(R, \cdot)$, $(C, +)$, $(C, \cdot)$ are monoid.

The structure $(Z, +)$ $(Z, \cdot)$ $(Q, +)$ $(Q, \cdot)$

$(R,+)$, $(R,\cdot)$, $(C,+)$, $(C,\cdot)$ are monoids.

(ii) The structure $(N,+)$ is a Semi-group but not a monoid, since the additive identity (i.e. 0) does not exists in N.

Group

Let G be a non empty set and '*' be a binary operation defined on G then the structure $(G,*)$ is said to be a Group if the following axioms are satisfied.

(i) Closure Property $a*b \in G \;\forall\, a,b \in G$

(ii) Associativity - If $a, b, c \in G$ then
$(a*b)*c = (a*b)*c \quad \forall\, a,b,c \in G$

(iii) Existence of identity — There exists an element $e \in C$ such that

$$a*e = a = e*a \quad \forall\, a \in G$$

e is called identity of * in G.

(iv) Existence of inverse. For each element $a \in G$ there exist an elem

$b \in G$ such that

$$a * b = e = b * a$$

The elt b is called the inverse of element a with respect to '$*$'

and we write

$$b = a^{-1}$$